A multi-objective optimum path algorithm for passenger pre-trip planning in multimodal transportation networks.

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ABSTRACT

This paper aims to present a multi-objective optimum path algorithm for passenger pre-trip planning in multimodal transportation networks. The process of identifying the feasible paths accounts the delays of the different modes use and of the switching terminals. This algorithm is designed to constitute a linking component to an integrated web based information gateway, aiming to provide information to its user through the internet for trips within Greece using public transport. The idea is to make a process for the system which identifies feasible paths according to compatibility of different modes, links and users preferences. The multi-objective linear programming model that corresponds to this process is here presented in order to prove the efficiency of the algorithm. The implementation of the proposed algorithm is also presented. After the implementation it is being coded and computationally tested in realistic size network. The computational complexity of the algorithm is proven polynomial.
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INTRODUCTION
The problem of trip planning in a system with competitive public transport means is subjective since the optimum route can be defined with several ways. Ideal is to have the ability of knowledge of all available ways to make a trip according to your needs or expects and then decide which route is optimal for you. The algorithm was structured in this exact concept. The need for the proposed algorithm became apparent in the context of a funded research project from the General Secretariat for Research and Technology of the Ministry of Development of Greece. The project aims to provide real time trip planning information taking into consideration all available means of interurban and urban public transport. The main issue is to create an algorithm that can take cognizance of user requests about mode preference and other factors such as cost, duration and delays at terminals, departure time of links and according to those produce the optimum sequence of routes. Also the need of providing alternative ways for the trip was desirable. In order to provide flexibility to the system the structure of the model is based on the following assumptions.

The system offers to the user the possibility to set the optimum route formed by various parameters, such as desired travel departure time, duration, type of transport means, total cost, maximum number of transits and waiting time. Trying to exploit in the best possible way the information provided by the user, it was considered appropriate to provide also the possibility to define priorities in the parameters in order to draw an individualised optimal route. At the same time the system has in its disposal the information of all mass transport routes in the network of interest (e.g. Greece) and for all competitive transport means (airplane, train, ship). This information is provided through internet from related service providers for each user’s request in order to have the availability and cost of the appropriate routes which can create the Origin–Destination connection. Taking into consideration transport network particularities and the integrated system’s objectives the tool that will correspond to the problem was inquired, for managing the O–D route connection where transit is required hubs having the ability to draw movements are used. In order to take in consideration this methodology a database was developed that has all the necessary information about hubs. So for each city that can be either origin or destination their hubs are defined at three levels: national, local and regional. In section 3 it is shown that this decomposition strategy is very helpful to build the search network for each O – D trip.

To develop an algorithm that would address the needs of the users as identified above, an extensive literature review has been performed to assess the available approaches in previous publications. Modesti and Sciomachen (9) presented probably the most influential approach dealing with the problem of finding the multi-objective shortest paths in multimodal urban transportation networks but they did not consider the delays at switching nodes. On the other hand, Ziliaskopoulos and Wardell (6) proposed an intermodal optimum path algorithm according to the principles of dynamic programming which considers the arcs travel time and
switching delays but defined as a least time optimum path. Bielli et al. (12) recently introduced a tool for finding the shortest path in a multimodal travel system using GIS- transit planning. A similar approach was presented by Zografos and Madas (7) who consider the shortest path in terms of least time shortest path or least number of mode transfers or mode or route preference. Finally, Berube et al. (8) consider the problem of time dependent shortest path through a fixed sequence of nodes. After reviewing the research of Ehrgott et al. (1-3) we focused on the multiobjective approach of our problem. For a review of multiobjective combinatorial optimization we referred to (4,5). Osman et al. (20) created a genetic algorithm applicable to multi-objective routing problems (MORPs) according to the principles of dynamic planning. Martins and Santos (21, 11) created a labeled algorithm for the multi-objective optimum path problem. Another problem approach was found in the work of Zhoo and Yen (22), who also proposed the use of a genetic algorithm for finding the multi-objective minimum spanning tree problem, finding a set of Paretos optimal solutions by the ideal point. Other researchers that studied the problem were Narula and Ho (23) with the degree - constrained MST, Ishii et al. (24) with the stochastic MST, Bertsimas (25) with the Probabilistic MST and Xu (26) with the quadratic MST. The added value of the algorithm that is presented consists in the use of multiobjective optimal routing according to users request and time-dependent routes of competitive modes. For a review of well known algorithms we referred to (13-19).

In this paper, a mathematical programming formulation is introduced which computes the optimum multimodal - intermodal path on transport network accounts for connections between modes and local transit options in section 2. In section 3 the implementation of the algorithm is being analyzed and additionally the combinational complexity of the phase of building the search network is proven polynomial. Finally in section 4 the conclusions of this work are discussed.

MATHEMATICAL PROGRAMMING MODEL

In this section the mathematical programming model that corresponds to the above mentioned requirements is introduced. First of all, the problem is considered as a classical problem of operational research and it is defined as a mixed assignment and transshipment problem. A multi – objective approach is used in order to provide the bi-criteria shortest path. First, the basic notation used in this paper is introduced. It is denoted with:

- **O** the set of cities that a trip originates
- **D** the set of cities than a trip terminates
- **H = O ∩ D** the set of hub cities
- **M** the set of modes
- **L** the set of links between i → j using mode m
- **L** the set of urban links within terminals with different mode in a city i ∈ H.

A transit network can be viewed as a connected directed graph. From every terminal with mode m of every city i ∈ O to every city j ∈ D where O = {the set of
cities that a trip originates} and \( D = \{ \text{the set of cities that a trip terminates} \} \) the following binary variables are defined:

\[ x_{i,j}^{m,l} : \text{decision variable which denotes the route from city } i \in O \text{ to city } j \in D \text{ with mode } m \in M \text{ and link } l \in L. \]

\[ y_{i,m}^{m_2,l} : \text{decision variable which denotes the urban route in city } i \in H \text{ which connects the terminal from mode } m \text{ to mode } m_2 \text{ using the } l^{-th} \text{link}. \]

For both cases when the decision variables equal to 1 means that the selected route will participate in the optimal path.

For each arc \((i,j,m,l)\) the following coefficients of \( x_{i,j}^{m,l} \) are considered to comprise the necessary information for the routes:

\[ T_{i,j}^{m,l} : \text{denotes the departure of link } l \text{ from city } i \text{ to } j \text{ with mode } m. \]

\[ c_{i,j}^{m,l} : \text{denotes the cost for the specific route } x_{i,j}^{m,l}. \]

\[ t_{i,j}^{m,l} : \text{denotes the duration of link } l \text{ from city } i \text{ to } j \text{ with mode } m. \]

Additionally for each \( i \in H \) that belongs to cluster hub the following coefficient of \( y_{i,m}^{m_2,l} \) are defined to comprise the necessary information for the urban links:

\[ T_{i,m}^{m_2,l} : \text{denotes the departure time of the urban link } l \text{ in city } i \text{ from terminal of mode } m \text{ to terminal of mode } m_2. \]

\[ c_{i}^{m,m_2} : \text{denotes the fare for the specific route.} \]

\[ d_{i}^{m,m_2} : \text{denotes the route duration in city } i \text{ from terminal } m \text{ to } m_2. \]

\[ h_{i}^{m,m_2} : \text{denotes the routes headway in city } i \text{ from terminal } m \text{ to } m_2. \]

For every terminal which serves mode \( m \) it is defined

\[ \tau_{m} : \text{to signify the minimum required arrival time for boarding on its routes } \forall j \in O. \]

\[ \tau_{m,j} : \text{to signify the maximum required transport time from a terminal gate to another for each } \forall j \in H. \]

\[ \tau_{m,j} : \text{to signify the maximum required transit time from the terminal to the nearest stop to board on an urban linkage in a hub for each } \forall j \in H. \]

\( s_1 : \text{target for goal of total cost} \)

\( s_2 : \text{target for goal of total trip duration} \)

\( d_{i}^1 : \text{overachievement of goal } i \ (i = 1 \text{ for cost, } i=2 \text{ for duration)} \) (positive deviation)

\( d_{i}^- : \text{underachievement of goal } i \) (negative deviation)

As mentioned above, the mathematical model is a multi-objective linear integer program (LIP.) that aims to take into consideration the totality of the objective functions. In the linear as well as in the multi-objective LIP, if the set constraints are not compatible there are no feasible solutions for the problem. Nevertheless, in the multi-objective linear programming a certain feasible solution that simultaneously accounts for all competing objectives is not always expected. Its aim is to find a solution that satisfies the system’s constraints and to be as close as possible to the
optimum objective functions values. Thus, the choice has been made to focus on a particular goal programming methodology. All objective functions are turned to system constraints introducing the declination variables from the goals set by the user. Henceforth, the problem objective function will be to minimise the declinations. The feasible space is defined by three types of constraints:

(i) **The time compatibility constraints of the links:**

In case of origin node the arrival time of the selected route should lie in between the desired time interval set by the user. \( T = \{ \text{desired travel departure time}\} \), \( \alpha = \{ \text{maximum declination from } T \} \). This means, that the following inequality should be satisfied:

\[
T - \alpha \leq \sum_{j \in D} \sum_{m \in M} \sum_{l \in L} T_{r,j}^m \cdot x_{r,j}^m \leq T + \alpha \quad \text{Where } j \neq r \text{ and } r \text{ the route origin node}
\]  

(1)

In case of a hub node constraint that would allow transit either at the same terminal or selecting terminal change should be considered. Waiting time due to transit should not exceed the maximum acceptable by the user waiting time denoted by coefficient \( T \). This means that the following inequality should be satisfied:

\[
\sum_{k \in D} \sum_{m \in M} \sum_{l \in L} T_{j,k}^m \cdot x_{j,k}^m - \sum_{m \in M} \sum_{i \in O} \sum_{l \in L} \left( T_{i,j}^m + t_{i,j}^m \right) \cdot x_{i,j}^m \leq T + \alpha \quad \forall j \in H
\]  

(2)

Clearly the difference between sequential roots should be in such an order to allow transit either at the same terminal or switching terminals in the hub. So the following inequality should be held:

\[
\sum_{k \in D} \sum_{m \in M} \sum_{l \in L} T_{j,k}^m \cdot x_{j,k}^m \geq \sum_{m \in M} \sum_{i \in O} \sum_{l \in L} \left( T_{i,j}^m + t_{i,j}^m \right) \cdot x_{i,j}^m
\]  

(3)

Additionally the arrival time of a route in a hub city should be less than the departure time of the posterior route. So the following inequality should be considered:

\[
\sum_{k \in D} \sum_{m \in M} \sum_{l \in L} T_{j,m}^m \cdot y_{j,m}^m \geq \sum_{m \in M} \sum_{i \in O} \sum_{l \in L} \left( T_{i,j}^m + t_{i,j}^m \right) \cdot y_{i,j}^m
\]  

(4)

For the selection of the time conventional urban link in a hub in case of switching mode the next inequality should be held:

\[
\sum_{m, m2 \in M} \sum_{l \in L} T_{j,m}^m \cdot y_{j,m}^m \geq \sum_{m \in M} \sum_{i \in O} \sum_{l \in L} \left( T_{i,j}^m + t_{i,j}^m \right) \cdot y_{i,j}^m
\]  

(5)

(ii) **Network constraints:**

From origin city only one route is selected and at destination city only one route arrives. So

\[
\sum_{j \in H} \sum_{m \in M} \sum_{l \in L} x_{r,j}^m = 1 \quad \text{where } r \text{ the origin node of the trip}
\]  

(6)
So \[ \sum_{i=O}^{\text{dest}} \sum_{m=1}^{M} \sum_{l=L}^{l} x_{i,s}^{m,l} = 1, \text{ where } s \text{ destination node of trip} \] (7)

Also for each \( i \to j \) connected cities \( 0 \leq \sum_{m=1}^{M} \sum_{l=L}^{l} x_{i,j}^{m,l} \leq 1 \) (8)

For each hub the transit process balance should be ensured since it constitutes a transshipment station. So the sum of inflows to the station should be equal to the sum of its outflows. To ensure the right sequence of urban and suburban links the following inequalities hold:

\[ \sum_{m=1}^{M} \sum_{l=L}^{l} y_{i,m}^{l,m} = \sum_{k \in D} \sum_{l=L}^{l} x_{j,k}^{l,k}, \forall j \in H, \forall m \in M \] (9)

\[ \sum_{m=2}^{M} \sum_{l=L}^{l} y_{i,m}^{l,m} = \sum_{k \in D} \sum_{l=L}^{l} x_{j,k}^{l,k}, \forall j \in H, \forall m \in M \] (10)

(iii) Constraints from transforming the objectives to constraints:

Introducing the declination variables which represent the positive and negative declination of the objective function (total route cost, duration) from the target price set by user.

- Objective function concerns the total cost of the suggested route

Assuming the fare price for the connection of city \( i \) with city \( j \) from terminal \( m \) by route \( l \) is \( c_{i,j}^{m,l} \), as well as the urban connection cost in city \( j \) of terminal with mode \( m \) with terminal with mode \( m2 \) is \( c_{j}^{m,m2} \), the total O-D route cost will be:

\[ \sum_{i \in O} \sum_{j \in D} \sum_{m=1}^{M} \sum_{L=1}^{L} c_{i,j}^{m,l} x_{i,j}^{m,l} + \sum_{j \in H} \sum_{m=2}^{M} \sum_{L=1}^{L} c_{j}^{m,m2} x_{j,m}^{m,m2} \]

Introducing the declination variables \( d_{1}^{+}, d_{1}^{-} \) which represent the positive and negative deviation of the objective function (total route cost) from the target price set by user \( s_{1} \). The declination variables are set to satisfy the attributes \( \geq d_{1}^{+} d_{1}^{-} \geq 0 \) and \( d_{1}^{+} \times d_{1}^{-} = 0 \). Therefore the objective function is converted to the following constraint.

\[ \sum_{i \in O} \sum_{j \in D} \sum_{m=1}^{M} \sum_{L=1}^{L} c_{i,j}^{m,l} x_{i,j}^{m,l} + \sum_{j \in H} \sum_{m=2}^{M} \sum_{L=1}^{L} c_{j}^{m,m2} x_{j,m}^{m,m2} + d_{1}^{+} - d_{1}^{-} = s_{1} \] (11)

- Objective function concerns the total duration of the suggested route

We find the total duration of suggested route by the arrival time at the destination if we subtract the origin departure time and add the required arrival time at the selected origin terminal for transit to one of its routes. Thus, it is given by the relation:

\[ \sum_{l=1}^{L} \sum_{m \in M} \sum_{j \in \text{dest}} (T_{i,j}^{l,m} + t_{j,i}^{m}) x_{i,j}^{m,l} - \sum_{l=1}^{L} \sum_{m \in M} \sum_{i \in \text{orig}} (T_{i,j}^{l,m} + t_{j,i}^{m}) x_{i,j}^{m,l} + \sum_{l=2}^{L} \sum_{m \in M} \sum_{i \in \text{orig}} (T_{i,j}^{l,m} + t_{j,i}^{m}) x_{i,j}^{m,l} \]

Introducing the declination variables \( d_{2}^{+}, d_{2}^{-} \) which represent the positive and negative declination of the objective function (total route duration) from the target price set by user \( s_{2} \). The declination variables are set to satisfy the attributes \( \geq d_{2}^{+} d_{2}^{-} \geq 0 \) and \( d_{2}^{+} \times d_{2}^{-} = 0 \). Therefore the objective function is converted to the following constraint.
The objective function will be formed depending on the priorities set by user $P_k$ for $k = 1, 2$ as well as on the weight assigned $w_{k,i}$ to underachievement or overachievement at priority $P_k$ for $k,i = 1, 2$. The objective function of the mathematical model for the linear goal programming problem can be stated as follows:

$$\min \sum_{k=1}^{2} P_k \left( \sum_{i=1}^{i} \left( w_{k,i}^+ d_i^+ + w_{k,i}^- d_i^- \right) \right)$$  (13)

The multi-objective LIP consists of the objective function (13) subject to constraints (1) – (12).

Trying to prove the efficiency of the suggested model, which is analyzed in this section, of managing the real problem of finding the optimum path with respect to users request as networks properties in multimodal transportation network it seems to be wise to solve it with well known software of linear programming. The development of the proposed model took place in AMPL which is a well known modeling language for mathematical programming. The separation of model and data was mainly the reason of this selection so it could be possible to use the produced model to any data set thus to any network. Several cases are tested in the AMPL model in order to check the inequality of the optimum path of the proposed algorithm. As mentioned above the system demands were not only the optimum routes but the feasible ones as well. Thus the mathematical programming model was developed to evaluate the optimality of the solution that the algorithm outputs.

**ALGORITHM IMPLEMENTATION**

Since the goal was mainly to manage as dynamically as possible the process it has been chosen to create an alternative algorithm. It is clear that the linear model dimension in real data is also the main factor that prohibits the use of the Simplex method for resolving the problem. Other factors that lead to this choice were the fact that the system wanted to suggest more than one route as well as to offer the user the possibility to form his own final choice by enabling the change of hers/his primary goals. The implementation of the process is separated in tree phases. In the first phase the search network is created in order to continue with the second, in which the area of feasible O-D pairs is formed, while in the third the individualised optimum route is formed off the system’s feasible solutions area.

The separation of the user profile constrains that will take part in the phases is given above:

Constrains that participate in both first and second phases:
  a. desired travel departure time, setting also its declinations,
  b. maximum transit waiting time,
  c. maximum number of transits,
  d. type of transport means.

Slack Constrains that participate in the third phase:
  a. total route desired duration,
  b. total route cost.
The area of all feasible routes is basically formed in the first two phases of the algorithm always respecting the restricted constraints set by the user. The solutions classification takes place in the third phase and is according to the goals and their priorities. The structure of the algorithm is shown in figure 1. In order to explain the procedures the three phases are explained.

FIGURE 1: Logical flow diagram of the algorithm.
Decomposition of search space

Despite the fact that the system has in its disposal all the information concerning trips of available transport means, it is considered necessary to form the base network of each O-D trip before the beginning of the feasible trips search procedure. In this phase, having the selected O-D trips by the users and the maximum number of transhipments allowed for the user’s trip planning, the network is formed at city level. As mentioned before, a database containing the hubs for each city has been created. In order to take into account the spatial placement of the hubs, they have been defined at three levels: national, local and regional. The hubs, as these have been defined in the database, are formed by cluster-origin and cluster-destination. The union of these two sets comprises the space of the hubs for each problem. Each cluster may have more than one hub that may belong to the same or to different levels. Their arrangement is done according the level they belong to. This means, that a trip is generated from an origin to a destination, where first the origin hubs at local level, then origin hubs at regional and national level are inserted and after the destination hubs at national, regional and finally local level. In the following table the selected trips are presented.

TABLE 1: Hub matrix where + represents connection and – represents no connection.

<table>
<thead>
<tr>
<th>Origin's Hubs</th>
<th>Origin</th>
<th>Local Hub</th>
<th>Regional Hub</th>
<th>National Hub</th>
<th>Destination</th>
<th>National Hub</th>
<th>Regional Hub</th>
<th>Local Hub</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local Hub</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Regional Hub</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
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<td>+</td>
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</tr>
<tr>
<td>National Hub</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
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<td>+</td>
<td>+</td>
</tr>
<tr>
<td>National Hub</td>
<td>-</td>
<td>-</td>
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<td>-</td>
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<td>Regional Hub</td>
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<tr>
<td>Local Hub</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>
Proposition 1: For n number of hubs which belong to different spatial levels, the number of alternative ways that O-D can be connected by making maximum k transhipments is \( \sum_{k=0}^{K} (n-k+1)^k \).

Proof:
Suppose that you want to impede a number of nodes into two nodes, and they can connect according to table 1, the possible way can be numerate as followed:
- For zero number of transfers there is only one way to establish a connection.
- If number of transfers equals to one there are n way for a possible connection.
- If number of transfers equals is two there are \((n-1)^2\) way for a possible connection.
- So for k number of transfers there are \((n-k+1)^k\) way for a possible connection.

According to the additional principal of enumeration the total count of different ways will be the sum of the above enumerations. Thus the number of alternative ways to connect O – D under those circumstances is \( \sum_{k=0}^{K} (n-k+1)^k \).

Proposition 2: For n number of hubs which belong to common spatial levels, the number of alternative ways that O-D can be connected by making maximum k transfers is \( \sum_{m=0}^{k} \frac{n!}{(n-m)!} \).

Proof:
Suppose we are interesting to numerate the possible ways to connect those two nodes with n different nodes with maximum k transfers all possible combinations should be counted as in the following table:
- For zero number of transfers there is only one way to establish a connection.
- If number of transfers equals to one there are n way for a possible connection.
- If number of transfers equals is two there are \(n(n-1)\) way for a possible connection.
- So for k number of transfers there are \(n(n-1)(n-2)\ldots(n-(k-1))=\frac{n!}{(n-k)!}\) way for a possible connection.

According to the additional principal of enumeration the total count of different ways will be the sum of the above enumerations. Thus the number of alternative ways to connect O – D under those circumstances is \( \sum_{m=0}^{k} \frac{n!}{(n-m)!} \).

Proposition 3: Algorithm computational complexity is less than polynomial degree \( O(n^k) \), where k denotes the maximum transfers and n the number of hubs that exist in the network.

Proof:
Because the following inequality holds
\[ \sum_{m=0}^{k} (n - m + 1)^{m} \leq \sum_{m=0}^{k} \frac{n!}{(n - m)!} = \sum_{m=0}^{k} n^{(m)} \] and the number of ways that O–D can be connected when k of n hubs are used, is maximum when they belong to common spatial levels. Additionally \[ \frac{n!}{(n-k)!} = \Delta_{k}^{n} = (n)_{k} = n^{(k)} \] which is called falling factorial polynomial and it can be written by using the Stirling numbers of the 1st kind as a polynomial
\[ n^{(k)} = S_{1}^{(k)} n + S_{2}^{(k)} n^{2} + S_{3}^{(k)} n^{3} + \ldots + S_{k}^{(k)} n^{k} = \sum_{i=1}^{k} S_{i}^{(k)} n^{i} \]
where \( S_{i}^{(k)} \) Stirling number of the first kind. Thus \[ \sum_{m=0}^{k} S_{m}^{(k)} n^{i} \Rightarrow O(n^{k}) \] and the computational complexity of algorithm in the level ordinance cities is \( O(n^{k}) \).

For the achievement of the procedure of the inserted hubs arrangement the subroutines “Hub comparer”, “HandleModeDoubleHubs” and “MakeHubMatrix” have been created, which respectively compare the two sets (Origin Hubs and Destination Hubs), serialize them, correct common registries for origin and destination and create a table in the form of Table 1. The “BuildPseudoLink” procedure creates the network from the table that has already been created. This procedure extracts the reference network on city level. By using this extracted information from the user’s profile concerning the transport means of her/his choice and the city terminals which have been defined by the former procedure, the network on terminals level is formed.

The complexity of the reference network on terminals level has been studied and it has been proved that the number of ways that O-Ds can be connected when there are \( n \) available hubs, \( m \) available terminals and at most \( k \) possible transhipments has an upper limit which is equal to \( \sum_{k=0}^{k} m^{k+1} n^{(k)} \).

It has been found that the complexity is significantly influenced by the user’s requests and by the database which contains the information concerning the hubs.

**Calculate feasible paths**

For the reference network that has been formed in the above mentioned procedure of the algorithm, available tickets, information regarding urban connections and points of transhipment are being searched. For every terminal there is information regarding costs, travel times and time of departure for every available transport mean. Also, information regarding time penalties for boarding, terminal transhipment and transhipment with change of transport means – that is transition to the closest point of boarding on urban connection – is given. For the heuristic procedure of the computation of feasible routes the subroutines “Run Trip” and “Run Terminal” have been created, where the first one calls the second for every terminal of the network. Compendiously, the procedure that is being followed is as follows:

The routes which satisfy the time limitations of the user for every connecting terminal of the network are being searched in every terminal and used to make the paths. The time limitations are formed with respect to the terminal’s location. For the origin
terminal the time limitation is: \( T - \alpha \leq T_{ij}^{m,l} \leq T + \alpha \) (1), where \( T \) represents the desired time of departure and \( \alpha \) the search time-window. For the terminals of the hubs:

- for extra-urban connections: \( T_{ij}^{m,l} + d_{ij}^{m,l} + \tau_{m,j} + \tau_{m} \leq T_{jk}^{m,l} \leq T \) (2) where \( T_{ij}^{m,l} + d_{ij}^{m,l} \) is the time of arrival in the terminal station \( j \) from \( i \) by the transport mean \( m \) and the \( l \) route.

\( \tau_{m,j} \) the maximum time needed for disembarkation at the station and transition to other gate of the station

\( \tau_{m} \) the minimum required time for boarding on routes of transport mean \( m \).

\( T_{2} \) the maximum waiting time for transshipment

- for urban connections: \( T_{ij}^{m,l} + d_{ij}^{m,l} + \tau_{m,j}^{*} + \tau_{m,j}^{*} + \tau_{m,j}^{*} + h_{m,m2}^{*} \) (3)

where \( T_{ij}^{m,l} \) is the departure time of the urban connection of two terminals which serve different mode

\( \tau_{m,j}^{*} \) the maximum time required for the transition from the terminal station to the closest station which provides boarding on urban transport means.

\( h_{m,m2}^{*} \) the headway of the urban connection

Compendiously, during the RunTerminal subroutine the routes that satisfy the conditions depending on the location (node id) are found and for each one of them a back up of the information up to this point is made. Following this route until the terminal station the total costs and the total travel times are computed and a control of the location is performed. If the route ends at the destination, it is being saved for the next phase. Otherwise the time compatibility control procedure is repeated. Overall, starting from the origin, the routes that satisfy condition (1) are found, for each one of them the path to the next city (either hub or destination) is created, in this city the time compatibility control procedures are executed with the same mode (2) or with mode change through urban connection (3) where the procedure is completed when reaching the destination. Analytically the procedure of computing feasible paths is defined below.

**Identify optimum path through criteria selection**

For the achievement of the alternative objective functions procedure, which can be set in the problem of optimum path, the respective criteria of ranking of paths have been created. This provides the optimum path and the alternatives as follows:

**StepI – Convert trips**

After the completion of the procedure of the connecting paths which satisfy the time limitations of the user regarding transhipment and time of departure for the desired transport means, the set of all the feasible paths of the trip planning problem has been created. For the feasible paths the following relationships define:

- total travel time

\( \tau_{m} + (T_{1,s}^{m,l} + d_{1,s}^{m,l}) - T_{r,j}^{m,l} \), where
\( \tau_{m} \): the required boarding time for routes of mode \( m \) from the origin

\( T_{m,l}^{i} \): the time of departure from the origin

\( (T_{i,s}^{m,l} + d_{i,s}^{m,l}) \): the time of arrival to the destination

- total travel costs

\[ \sum_{m,l,i \in P} c_{i,j}^{m,l} + \sum_{i,d \in H} c_{i,m}^{m,l} \], where

\( c_{i,j}^{m,l} \): the costs of extra-urban routes

\( c_{i,m}^{m,l} \): the costs of urban routes

**Step 2 – ‘Sort trip’**

For every feasible path in a table, the information containing the total travel costs and total travel time is saved, which then are ranked according to the following criteria:

1. Criterion of minimum duration or cost with or without goals defined.
2. Criterion of minimum duration or cost with priorities and goals defined.
3. Criterion of minimum duration or cost with weights and with or without goals defined.

Select criterion and according to the selected criterion the sorting of route will take place. The first route will declare the optimal route. Thus the vector \((i, g_i, h_i)\) where \(g_i\) : total duration accounted the delays, \(h_i\) : total cost for interurban and urban links that participate to the route \( i \). The \( s_1, s_2 \) are defined to represent the goal that the user requested. It should be \( g_i \leq s_1 \); \( h_i \leq s_2 \). To sort the cases the user can use the below criterions. Analytically the process for each criterion is defined below.

**Criterion of minimum duration or cost with or without goals defined**

In this criterion the shortest path will be declared by sorting cases either by total cost or by total duration. It can be done even if the user specifies a target value of the expected overall cost (fees or duration).

**Criterion of minimum duration or cost with priorities and goals defined**

In this criterion the user specifies the goals and the priorities to the objectives and the optimum path will be declared where the first priority objective should be ensured and then minimize the variation of the target value of the second’s priority goal. Thus when the first goals is ensured the order is based on the second one. If the satisfaction of the first goal is uninsured the optimal route concludes to\([\min(d_{j,s})]\). The first route is the one that variants less from the first priority goal.

**Criterion of minimum duration and cost with weights and with or without goals defined**
In case that goals and weights are defined a generalized objective function is defined based on variations of the goals and the weights that are assigned to them. The optimal route is the one that minimizes the combination of the variations of $g_i, h_i$ from $s_1, s_2$.

**COMPUTATIONAL EVALUATION OF IMPLEMENTED ALGORITHM**

In order to estimate the computational complexity of the implementation of the proposed algorithm several cases have been tested and for each one the CPU time that was required for the final results has been calculated. Also the number of links that structures the network has been measured, the maximum waiting time that the user has requested and the goal trip duration she/he gave.

The following figure shows that the CPU time is linear dependent to the number of links that participate in the search network of any O – D connection.

![FIGURE 2: Graph of the average CPU time (in seconds) related to number of links.](image)

Also average CPU time is nearly linear related to the maximum waiting time that a user has determined in order to make an interchange in a hub as shown in the following figure.
FIGURE 3: Graph of the average CPU time (in seconds) related to waiting time.
Also average CPU time is nearly linear related to maximum trip duration that a user has determined. From the above figures we can see that the average CPU time increases with the travel duration to an exact value and then it remains constant. This is reasonable because the other constrains probably bound the process.

FIGURE 4: Graph of the average CPU time (in seconds) related to maximum duration.

CONCLUSIONS
The present paper presents an algorithm which identifies feasible connected routes for passenger trip planning in a national network with competitive modes based on each user's request and it also extracts the optimum path based on priorities and target values of duration and cost of the trip. This algorithm was designed to constitute a linking component to an integrated web based information gateway, aiming to provide information to its users through the internet for trips within Greece using public transport. The main issue for the system was to give the ability to users to set significant factors that leads to the decision of which path is more likely to her/his needs or requests. Additionally the integrated system has in its disposal all the available competitive mass means. Thus it has become important to
create a tool that can manage complex user’s requests and available routes in order to extract the optimum combination of competitive routes for planning a trip. The algorithm was designed to simulate the process of managing the critical issues which lead to the final decision for trip planning in a national network. The multi – objective linear programming model was structured for checking and certificating the optimality of the proposed optimum path. In the process of validation of the proposed algorithm we prove that the flexibility of the selected constrains impacts the computational complexity linear as it was predictable and logical. In order to have the ability to compute combinations of selected routes we focused on a decomposition strategy. A critical issue to this aspect was the hub selection of each city which can constitute either an origin or a destination city. The research of hub selection is very important for the compatibility and viability of multimodal transportation systems. Such research can lead to identification of various parameters which increase the compatibility of services and fees. Also such research can make this algorithm applicable, with some variations maybe, to European network levels. It is believed that our proposal could be seen also as a very promising approach for the analysis of the user’s preferences in their mobility within urban and interurban transportation networks and consequently for an implementation for public transportation offers.

REFERENCES


